5: Emission (Computed) Tomography

1. What is a tracer?
2. Why is collimation necessary and what are its consequences?
3. How are the effects of attenuation taken into account?
4. What is the principle of x-ray detection?
5. How are scintillation photons converted to an electrical signal?
6. How can scattered photons be eliminated?

After this course you
1. Understand the reason for collimation in imaging $\gamma$-emitting tracers and its implication on resolution/sensitivity
2. Understand the implications of x-ray absorption on emission tomography
3. Understand the basic principle of radiation measurement using scintillation
4. Are familiar with the principle/limitations of photomultiplier tube amplification
5. Understand the use of energy discrimination for scatter correction

What is Emission Computed Tomography?

Until now: CT and x-ray imaging measure attenuation of incident x-ray

Emission tomography: X-rays emitted by exogenous substance (tracer) in body are measured

What is a tracer?
Exogenously administered substance (infused into blood vessel) that
(a) alters image contrast (CT, MRI)
(b) has a unique signal ($\gamma$ emitting)

$\rightarrow$ Emission computed tomography

<table>
<thead>
<tr>
<th>Typical tracers for emission tomography</th>
<th>[h]</th>
<th>[keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{99m}$Tc</td>
<td>6</td>
<td>140</td>
</tr>
<tr>
<td>$^{201}$Tl</td>
<td>73</td>
<td>70</td>
</tr>
<tr>
<td>$^{123}$I</td>
<td>13</td>
<td>159</td>
</tr>
<tr>
<td>$^{133}$Xe</td>
<td>0.08</td>
<td>81</td>
</tr>
</tbody>
</table>
What are the basic elements needed for γ-emitter imaging?

Collimator
Photomultipliers
Light guide
Scintillating crystal
Collimator

What are the basic elements needed for γ-emitter imaging?

- Photomultipliers
- Light guide
- Scintillating crystal
- Collimator
- Septa

5-2. How can directionality of x-rays be established?

**Collimation**

**Problem:** Photon detection alone does not give directionality.

**Solution:**

Consider one detector, assuming perfect collimation (and neglecting attenuation, see later).

\[ S(y) = \int_{-\infty}^{\infty} C_T(x, y) \, dx \]

Signal \( S(y) \)

\( C_T(x, y) \): tissue radioactivity

Radon transform

\[ S(y) \Rightarrow \text{Reconstruction as in CT} \]

Line of incidence (LOI)

Impact of collimation on resolution

Collimator
Thick (lead or tungsten) with thin holes
Select rays orthogonal to crystal

Collimation establishes direction of x-ray

\( x \)

\( y \)
How does collimation affect resolution?

It’s never perfect …

Perfect collimation, i.e. resolution?

\[
d/a \to 0 \\
\mu_{\text{collimator}} \to \infty
\]

Impossible to achieve (Why?)

Collimator resolution:

Two objects have to be separated by distance \(R\)

\[
R = \frac{d(a_e + b)}{a_e}
\]

\(a_e = a - 2 / \mu\)

(\(a_e\): imperfect septal absorption)

Septa penetration < 5% occurs when \(t_{5\%}\): 

\[
t_{5\%} = \frac{6d}{a - 3 / \mu}
\]

Price of collimation (resolution)?

Sensitivity!

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5-3. How to deal with attenuation of the emitted x-rays?

result of x-ray absorption in tissue

Signal measured from a homogeneous sphere \((C(x,y) = \text{constant})\)

Intensity distortion: Cause?

\[\text{Intensity distortion:} \quad T = \frac{n(D)}{N_0} = e^{-\mu D}\]

1. depends on object dimension and source location \((D = f(\text{object}))\)

Consider point source:

Attenuation depends on location of source in tissue

2. Photon energy \(\mu = f(E_x)\)

\(\mu_{\text{water}}(140keV) = 0.16 \text{cm}^{-1}\)

\(\mu = 0.693 / \mu \approx 4.5 \text{cm}\)

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What are the basic steps in attenuation correction?

**Attenuation correction procedure**

A. Estimated object geometry and estimated \( \mu(x,y) \) or measured \( \mu(x,y) \)

B. Transmission loss: \( T(\text{projection}) = f(\mu(\text{object}), \text{projection}) \)

C. Attenuation correction \( A(x,y) = \frac{1}{T(x,y)} \)

D. Corrected \( C_{corr}(x,y) = A(x,y) \cdot C(x,y) \)

Problem is prior knowledge needed for \( A \) (i.e. \( \mu(x,y) \))

Attenuation correction rarely applied!

How to simplify attenuation correction?

by measuring at 180° using geometric mean

**Problem:** Spatial dependence of correction

\[ D = x_1 + x_2 \]

Consider point source:

Signal \( (S_1) \)

\[ S_1(y) = C_T(x_1, y) e^{-\mu x_1} \]

Signal \( (S_2) \)

\[ S_2(y) = C_T(x_2, y) e^{-\mu x_2} \]

\( C_T(x,y) \): tissue radioactivity

**Solution:** Geometric mean of the two 180° opposite signals:

\[ \sqrt{S_1 \cdot S_2} = C_T(x, y) e^{-\frac{\mu D}{2}} \]

NB. This correction can be used in emission tomography for focal uptake (i.e. uptake limited to a specific region)

Measure at 180° simultaneously and take the geometric mean

→ attenuation correction depends only on dimension of object along the measured Radon transform
5-4. What is the principle of x-ray detection?
Collimation, followed by scintillation and amplification

Scintillator crystal
  e.g. Ti-doped SodiumIodide (NaI)

Photomultiplier Tube

\[ \gamma \text{-energy } \propto \# \text{ scintillation photons } \propto \text{Signal} \]

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What is Scintillation?

Sequence of events in scintillation crystal
1. Atom ionized by Compton interaction \( \rightarrow \) Electron-hole pair
2. Hole ionizes activator, electron falls into activator
3. Activator is deactivated by emission of Photons (10^{-7} \text{ sec})

Efficiency of scintillators
\[
\eta = \frac{\text{energy of scintillation light}}{\text{energy deposited}} \propto \frac{Tq_a}{W_{e-h}}
\]
\[ T = \text{energy transfer efficiency from excited ion to luminescence centre} \]
\[ q_a = \text{quantum efficiency of luminescence centre} \]
\[ W_{e-h} = \text{energy required to create one electron-hole pair} \]
What elements characterize scintillation materials?

Overview of some crystals

<table>
<thead>
<tr>
<th>Scintillator</th>
<th>Density (g/cm³)</th>
<th>Attenuation Coefficient (cm⁻¹ at 511 keV)</th>
<th>Light yield ph/keV</th>
<th>λ (µm)</th>
<th>τ (µs)</th>
<th>Z eff Refr. Index</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>CdWO₄</td>
<td>7.90</td>
<td>0.886</td>
<td>15</td>
<td>495</td>
<td>~10⁴</td>
<td>73</td>
<td>13%</td>
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<tr>
<td>Bi₄Ge₃O₁₂</td>
<td>7.13</td>
<td>0.964</td>
<td>8</td>
<td>480</td>
<td>300</td>
<td>51</td>
<td>1.85</td>
</tr>
<tr>
<td>(Y,Gd)₂O₃:Eu,Pr</td>
<td>5.9</td>
<td>0.503-0.637</td>
<td>13</td>
<td>610</td>
<td>~10⁶</td>
<td>59</td>
<td>66</td>
</tr>
<tr>
<td>Ga₂O₃:S,Pr,Ce,F</td>
<td>7.34</td>
<td>0.786</td>
<td>44</td>
<td>510</td>
<td>~10³</td>
<td>59</td>
<td>66</td>
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<tr>
<td>NaI:Tl</td>
<td>3.67</td>
<td>0.343</td>
<td>40</td>
<td>415</td>
<td>230</td>
<td>51</td>
<td>1.85</td>
</tr>
<tr>
<td>Gd₃SiO₄:Ce</td>
<td>6.71</td>
<td>0.704</td>
<td>7</td>
<td>430</td>
<td>300</td>
<td>59</td>
<td>66</td>
</tr>
<tr>
<td>Lu₂SiO₅:Ce</td>
<td>7.4</td>
<td>0.869</td>
<td>30</td>
<td>420</td>
<td>40</td>
<td>66</td>
<td>79%</td>
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<tr>
<td>Lu₃AlO₅:Ce</td>
<td>8.34</td>
<td>0.956</td>
<td>11</td>
<td>365</td>
<td>~17</td>
<td>66</td>
<td>79%</td>
</tr>
<tr>
<td>LuPO₄:Ce</td>
<td>6.53</td>
<td>0.735</td>
<td>17</td>
<td>360</td>
<td>25</td>
<td>66</td>
<td>79%</td>
</tr>
</tbody>
</table>

Requirements for scintillator

- High yield
- Good linearity
- Small time constant τ
- Transparent for scintillation light λ
- Good mechanical properties
- Refraction index close to 1.5

Most of the energy of the x-ray is lost as heat to lattice, see e.g. NaI(140keV)=40•140\ =5600 \text{photons at } \lambda \approx 400\text{nm}

\[ E_{400\text{nm}}[\text{keV}]=h\nu=1.2/\lambda [\text{nm}] \]

or <10eV/keV

5-13

5-5. How is the scintillation light converted to an electrical signal?

Photomultiplier tube (PMT) - Noiseless amplification

PMTs are bulky

How to increase resolution beyond PMT dimensions?
How to improve the spatial resolution of PMT? (Anger, 1964)

$$x = \frac{\sum x_k I_k}{\sum I_k}$$

Looks familiar? (see center of mass 1st year Physics)

5-5. How to discriminate scattered photons?

Most scattering is by Compton

$$E_f = \frac{E_i}{1 + E_i \left(\frac{1 - \cos \theta}{m_e c^2}\right)}$$

Measure $E_f$ to identify severely scattered photons.

<table>
<thead>
<tr>
<th>theta/Ei</th>
<th>100</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>99</td>
<td>138</td>
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<td>110</td>
<td>79</td>
<td>102</td>
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<tr>
<td>180</td>
<td>72</td>
<td>90</td>
</tr>
</tbody>
</table>

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What processes contribute to the Scintillation light spectrum?

Scintillation signal depends on x-ray energy.

Nal (Tl) scintillation peak for Cs-37: 662 keV

- Max. energy of the recoil electron (i.e. 662 keV photon scattered by 180°)
- Energy of 662 keV photon scattered by 180°
- Light from Compton events
- Secondary photons escaped from crystal

SPECT summary

Single Photon Emission Computed Tomography

1. Measurement of single photon emitters injected into subject
2. Collimation ensures x-ray directionality (⇒ backprojection)
3. Absorption is undesirable
4. Photon energies comparable to CT ⇒ SPECT-CT