1. What factors influence contrast in x-ray imaging?
   - Beam hardening
   - Sensitivity and resolution considerations
2. What influences CNR of x-ray imaging?
3. What is the fundamental basis for image reconstruction using x-ray absorption?
   - Radon Transform
4. How can x-ray images be reconstructed?
   - Sinogram
   - Backprojection vs. filtered backprojection
   - Central Slice Theorem

5. Examples & Summary

   After this course you
   1. Understand the consequences of the Bremsstrahlung continuum on image contrast
   2. Understand how Compton scattering reduces image contrast and how its influence can be reduced
   3. Are familiar with the Radon transform
   4. Understand the principle of matrix reconstruction and backprojection
   5. Understand the major mechanisms leading to CT contrast

4-1. What does absorption in the real world imply?

**Linear attenuation coefficient \( \mu \)**

\[
\begin{align*}
\mu & : \text{linear attenuation coefficient} \\
\text{Unit: [cm}^{-1}\text{]} 
\end{align*}
\]

If \( \mu \) is constant in \( x \)

\[
n(x) = N_0 e^{-\mu x}
\]

The measurement that is **wanted**: \( \mu(x,y) \)

What is **measured**: \( n(x) \)

Contrast is “well-defined” for monochromatic x-rays

\[
\ln \left( \frac{n(x)}{N_0} \right) = -\mu x
\]

(\( \mu \) for a homogeneous object of thickness \( x \))

But, \( \mu = f(E, Z, \rho) \)

Two consequences:
- Beam hardening
- Depth dependent contrast
What does the Energy Spectrum of an x-ray tube really look like?

filtered Bremsstrahlung and characteristic emission

\[ I_0 = \int_0^\infty i(E) \, dE \]

\[ i(E): \text{complex function} \]

Define effective photon energy \( E_{\text{eff}} \)

Minimal energy is zero, but:
Soft x-rays (low energy) are filtered by instrument

Hard x-rays (high energy)

Maximal x-ray energy is = kinetic energy of e\(^{-}\) (eV\text{cathode})

Interaction with orbital electrons

\( E_{\text{eff}} \) is increased by instrument (filtering of soft x-rays)

---

What is the consequence of energy-dependent absorption?

Beam Hardening - Effective energy depends on depth

A similar consequence arises in tissue:

Ideal:
Monochromatic x-rays
\((E_0(\lambda) = \delta(\lambda_0))\)

Reality:
Polychromatic, multienergetic \(i(E_0)\)

Absorption is not uniform with \( E_0 \)
- Contrast changes with large objects and depth
- Excessive radiation dose to superficial tissue

“Solution”: Reduce \( i(E_0) \) for soft x-rays
(e.g. 3mm Al eliminates 90% of 20keV photons)
4-2. How does x-ray scattering impact CNR?

Scattering increases with FOV (field-of-view) of irradiation
⇒ Further reduction of image contrast

Solution: Anti-scatter grid (collimator)

With \( E \), \( Q \)
Compton scattering
increased masking of object.

How is CNR quantified?

**Signal:**
\[ I(d) \propto I_0 \]
\[ I(d) \propto I_b \]

**Contrast:** \( \Delta I(d) \) due to \( \mu(d) \)
- \( \Delta I(d) \propto \mu_\text{C} \) produces reduced contrast
- Compton scattering: Antiscatter grid

CT intensity can be measured in absolute terms (CT-number)

Soft tissue: Typically has weak contrast (small Hounsfield units)

**CT-numbers of tissue in Hounsfield units (HU)**

\[ \text{CT number} = \frac{\mu - \mu_\text{water}}{\mu_\text{water}} \times 1000 \]

HU: attenuation normalized to water (=0)
- range from -1000 (air) to +3000 (bone and contrast agents)
- soft tissues: -300 to +100
4-3. What is the basis of image reconstruction?

The Radon transform

Given a certain beam intensity (no. of photons) $I$ at a given position $y$, $I(y_0)$, the beam intensity at $y + \Delta y$ is

$$I(y_0 + \Delta y) = I(y_0) e^{-\mu(y_0)\Delta y}$$

Recursive application to derive $I(y_0 + 2\Delta y)$

$$I(y_0 + 2\Delta y) = I(y_0) e^{-(\mu(y_0 + \Delta y)\Delta y + \mu(y_0)\Delta y)}$$

$$\lim_{\Delta y \to 0} I_{\text{detected}} = I(y_0) e^{-\int_{-\phi_0}^{\phi_0} \mu(x,y') dy'}$$

Considering a two-dimensional object:

$$I_{\text{detected}}(x) = I_0 e^{-\int_{-\phi_0}^{\phi_0} \mu(x,y') dy'}$$

$I_0$: intensity of incident x-ray beam

Radon transform $g(x)$

Definition

$$g(x) = \int_{-\phi_0}^{\phi_0} \mu(x,y') dy'$$

Radon transform of a point-like homogeneous object

Rectangular object

Radon transform of a circular object

Radon transform $= \text{projection of object}$

Trajectory:

$$R \sin(\phi_0)$$

Each point in space is uniquely represented by Amplitude $R$ and phase $\phi_0$ of sinusoidal trajectory in Sinogram (sic!): $(x,y) \rightarrow (R,\phi_0)$

Does each pixel have a unique trajectory?

Sinogram

Detector: is moved in circular motion around object (indicated by angular position $\phi$)

View A

View B
Can a CT image be constructed by Matrix inversion?

Decomposing an object into a 2x2 matrix requires a minimum of 4 measurements:

\[
I_j = I_0 e^{-(\mu_1 \Delta x + \mu_2 \Delta y)}
\]

\[
\ln(I_1/I_0) = -\mu_1 \Delta x - \mu_2 \Delta y
\]

\[
\ln(I_2/I_0) = -\mu_1 \Delta x - \mu_4 \Delta y
\]

\[
\ln(I_3/I_0) = -\mu_3 \Delta y - \mu_4 \Delta y
\]

\[
\ln(I_4/I_0) = -\mu_2 \Delta y - \mu_4 \Delta y
\]

Setting \(\Delta x = \Delta y\) yields a linear 2x2 inversion problem linking \(\mu_k\) to \(I_k\).

CT was introduced in 1970 ⇒ simple reconstruction algorithm!

4-4. What algorithm is adapted to 1970’s computing power?

Backprojection reconstruction

Basic reconstruction principle: Along the measured projection direction fill in each pixel constant numbers corresponding to the Radon transform (projection intensity).

Repeat for next orientation of the projection, sum the values in overlapping pixels.

Illustration with gray shades (point-like object):

2 Projections 4 Projections

\[
R_{2w}(x') = \int f(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy'
\]

\[
\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

8 Projections 16 Projections
Why does simple Backprojection have poor spatial resolution?

Backprojection has poor spatial resolution:

Reconstruction of a point-object falls off with $1/r$

The reconstruction falls off with $1/r$
(in analogy to the decrease of light intensity in 2D)

$$dx dy = dA = r \sin(d\phi) d\phi \approx d\phi d\rho$$

Number of rays (projections): constant with $d\phi$

But:

$$dA \propto \rho$$

$$\text{pixel size} = dx dy \propto dA = \text{const}$$

$$\Rightarrow \text{No. of rays} \propto 1/\rho$$

How can good image resolution be maintained?

Filtered Backprojection

Filtered Backprojection Example

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How is Backprojection linked to Fourier transform?

Central Slice Theorem

Image space

\[ M(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) e^{-2\pi i k_x x} e^{-2\pi i k_y y} \, dx \, dy \]

Final Image (CT: acquired data)

k-space (Fourier space)

\[ M(0, k_y) = \int_{-\infty}^{\infty} \mu(x, y) \, dx \]

FT of projection → 1 line in k-space

⇒ Reconstruction using FT:

MRI: Acquired Data

See also: Signals and Systems (SV)

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4-5. X-ray CT: Examples (Human)

Bone (calcification): bright (high absorption)

Imaging of mummified bodies

Air is dark

Different densities of tissue give intermediate results

Dislodged arrow head
CT: Examples (mouse)

3D CT scan of rodent spine treated with human mesenchymal stem cells (transduced with the human BMP-9 gene via an adenoviral vector) significant bone formation at the treatment sites (arrows)

13μm micro CT of mouse placenta vasculature

Micro-CT of mouse femor bone

CT: Summary

Main **contrast** is bone vs. soft tissue (or air) (calcium content i.e. \( \text{e}^\text{d} \)ensity \( \rho \))

Contrast agents (increase \( Z_{eff} \)) allow depiction of vessel architecture and lesions

**SNR and CNR:**
1. Intensity can be increased by cathode current
2. High spatial resolution possible (limited only by radiation dose in humans)
How have CT scanners evolved?

Generations of CT scanners

First Generation
- Parallel beam design
- One/two detectors
- Translation/rotation

2nd Generation
- Small fan beam
- Translation/rotation
- Larger no. of detectors

3rd Generation
- Multiple detectors
- Large fan beam

4th Generation
- Detector ring
- Large fan beam