2: Ultrasound imaging and x-rays

1. How does ultrasound imaging work?
2. What is ionizing electromagnetic radiation?
   - Definition of ionizing radiation
3. How are x-rays produced?
   - Bremsstrahlung
   - Auger electron

After this course you
1. understand the basic principle of ultrasound imaging
2. Are able to estimate the influence of frequency on resolution and penetration.
3. are capable of calculating echo amplitudes based on acoustic impedance;
4. know which parts of the electromagnetic spectrum are used in bio-imaging
5. know the definition of ionizing radiation;
6. understand the principle of generation of ionizing radiation and control of energy and intensity of x-ray production;

2-1. What are the main fates of US waves in matter?

1. Attenuation
   - Sound wave travels through the substance but loses energy $I(x)$
   - $I(x) = I_0 e^{-\alpha x f}$
   - Attenuation coefficient $\alpha$ [dB/(cm MHz)]
   - $\alpha$ is usually given in dB: dB=10log(I(x)/I_0)
   - [3dB=2fold increase in I(x): 10^0.3=2]
   - Unit conversion: k=ln10/10
   - Typically $\alpha$~0.5dB/(cm MHz)
   - → 6MHz signal will lose 3dB per cm of travel (2 fold loss in wave energy)

2. Refraction
   - Sound wave bends as it hits an interface at an oblique angle

3. Scatter
   - Sound wave dispersed in all directions

4. Reflection
   - Sound wave bounces back to probe
   - Reflection (echo formation) is key to imaging
What is the basic principle of US imaging?

**The basic principle** of imaging using sound waves:
1. Emit sound pulse (length \([1-5 \mu s]\) is a multiple of cycle time \(1/f\))
2. Measure time and intensity of echo
3. Reconstruct using known wave propagation velocity \(c\)

**Distance of tissue boundary from probe (transducer)**

**Ultrasound:** frequency \(f=1-20\text{MHz}\) (not 20kHz)

**Sound wave propagation velocity** \(c = \lambda f\)
- \(\sim 330\text{m/s (air)} = 0.33 \text{mm/\mu s}\)
- \(\sim 1.45-1.6 \text{mm/\mu s (tissue)} \Rightarrow (1\text{cm} - 7\mu s)\)

\((\text{increases with density } \rho, \text{ bone } \sim 4 \text{ mm/\mu s})\)

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What determines the resolution in US imaging?

**1. Resolution**
   - Increases with \(f\)

**2. Penetration** (cf. attenuation)
   - Decreases with \(f\)

**Wavelength** \(\lambda\) determines minimal resolution

**1. To have defined frequency:**
   - Pulse length = \(N/f = \lambda\)

**2. Separation of return echoes, e.g.**
   - \(\Delta T > 2 \text{ pulse length}\)
When does an acoustic echo occur?

Acoustic impedance and reflection ratio

### Acoustic impedance $Z$

**Definition:**

$$Z = \rho c \text{[kg/m}^2\text{s=rayls]}$$

### Amount of reflected wave energy

$$I_{ref} = I_0 R_I$$

At interface between objects with different acoustical properties

**Transmission**

$$T_I = 1 - R_I$$

**Probability of reflection + transmission is**

$$= 1:$$

### What are the reflection coefficients $R_I$ between tissues?

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<th>$R_I$</th>
<th>Fat</th>
<th>Muscle</th>
<th>Skin</th>
<th>Brain</th>
<th>Liver</th>
<th>Blood</th>
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<tr>
<td>Blood</td>
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<tr>
<td>Cranial bone</td>
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</table>

**Reflection by solid material**

e.g. bone-tissue interface

⇒ Shadow formation: ~45% of energy transmitted

$$T_I = 1 - R_I$$

**Dolphin fetus**

Fund BioImag 2016
2-2. What is the optimal choice of US frequency?

Resolution:
\[ \Delta x \text{ decreases with increasing frequency } f : \propto \frac{1}{f} \]
\[ \Rightarrow \text{Resolution} \propto f \]

SNR:
Signal returned from an echo-generating tissue interface at distance \( x \) from transducer:
\[ S(f, \alpha, x) = S_0 e^{-\alpha x f^2} R_{f_{0}} \]
\( f \): US frequency (experimental parameter)
\( \alpha \): attenuation coefficient (tissue parameter)

Find the optimal \( f \) ...
\[ \Rightarrow \text{Maximize } f \cdot S \]

\[ d\left( f S(f, \alpha, x) \right) = R_{f_{0}} \frac{d}{df} f e^{-\alpha f^2 x} \]

**SNR**

\[ \text{Resolution: } \Delta x \text{ decreases with increasing frequency } f : \propto \frac{1}{f} \]

**SNR:**
Signal returned from an echo-generating tissue interface at distance \( x \) from transducer:
\[ S(f, \alpha, x) = S_0 e^{-\alpha x f^2} R_{f_{0}} \]
\( x \): distance from transducer
\( f \): US frequency (experimental parameter)
\( \alpha \): attenuation coefficient (tissue parameter)

**Find the optimal \( f \) ...**
\[ \Rightarrow \text{Maximize } f \cdot S \]

**How critical is the choice of \( f_0 \)?**

\[ \frac{df}{df} \left( f e^{-\alpha f^2 x} \right) = 0 \]

\[ e^{-\alpha f^2 x} \left( 1 - f^2 \alpha x \right) = 0 \]

\[ f_0 = \frac{1}{2\alpha x} \]

The optimal frequency decreases with tissue depth and with increasing absorption.

**Ex. 3-D US Imaging & Contrast agents**

3D US Physical Principle:
1. the transducer is moved during exposure (linear shift, swinging, rotation)
2. received echoes are stored in the memory
3. the image in the chosen plane is reconstructed mathematically

**Contrast agents:** gas-filled Bubbles

Gas: most contrast (plus resonance and higher harmonic imaging) (see tiny Z \( \rightarrow \) total reflection, \( R_{f_{0}} \))

**Umbilical cord**
**How can Ultrasound detect moving blood?**

**Doppler effect**

Motion (Doppler): Frequency shift $f_D$ of moving tissue, results in shifted US frequency (demodulation for detection) (where is this also used?)

Doppler frequency shift $f_D$

$$f_D = \frac{2 f_0 v_0 \cos \alpha}{c}$$

- $c$: speed of US, e.g. 1500 m/s
- $v_0$: speed of source, e.g. 0.5 cm/s
- $f_0$: frequency of moving source, e.g. 5MHz
- $\alpha$: Rel. angle at which blood is moving

Example:

$$f_0 = 2 \cdot 5 \cdot 10^6 \text{ [Hz]} \cdot 0.5 \text{ [m/s]} / 1500 \text{ [m/s]}$$

$\sim 3 \text{kHz}$

$\sim 0.05\%$ of $f_0$

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**2-3. Basis of x-ray imaging**

useful relationships Electromagnetic radiation

$$c = \lambda \nu$$

$c = \text{speed of light} = 3 \cdot 10^8 \text{m/s}$

$$E = h \nu = \frac{hc}{\lambda}$$

$h = 2 \pi \cdot 10^{-34} \text{Js}$

$= 4 \cdot 10^{-19} \text{keVs}$

$1 \text{eV} = \text{energy of e}^-$ in acquired in 1V electric field

$$E = \frac{hc}{\lambda}$$

$= 1.2 \text{keV/nm}$
With which elements of matter does EM radiation interact mainly?

(in imaging mainly with electrons)

**Electron binding energy**

![Energy level diagrams for hydrogen and tungsten](image)

Binding energy
1. decreases with shell distance
2. increases with Z
(Why?)

Lowest K-shell binding energy:

\[ E_{K_{\text{min}}} = 13.6 \text{eV (}^1\text{H)} \]

\( h\nu > E_{K_{\text{min}}} \): ionizing
\( h\nu < E_{K_{\text{min}}} \): non-ionizing

**Electron (some useful constants)**

\[ m_e = \text{mass} = 9 \cdot 10^{-31} \text{kg} \]
\[ q_e = \text{charge} = 1.6 \cdot 10^{-19} \text{ C (As)} \]

Rest energy \( m_e c^2 = 511 \text{ keV} \)

**Ionizing radiation is above 13.6 eV**

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### 2-4. How are x-rays generated (scheme)?

Negatively charged cathode = electron source

Electrical current (filament current) heats up the cathode (why is that necessary?)

Electrons are liberated and accelerated by electric field (Energy of \( e^- = q_e \times V \))

Anode = metal target (tungsten)

accelerated electrons hit anode \( \Rightarrow \) generate X-rays

(tube current with voltage difference up to 150 kV)

**Intensity of beam = Power/Area**

1. Number of X-rays (proportional to tube current)
2. Energy of X-rays \( h\nu \) (proportional to voltage)
Emission of x-rays I: What is Bremsstrahlung?

Consider the interaction of e⁻ with stationary atom as collision:

\[ p_i = p_f + p_{\text{photon}} \]

Coulomb:

\[ a \sim q_e Z/m_e r^2 \]

\[ P_{\text{Brems}} = q_e^2 a^2/6\pi\varepsilon_0 c^3 \]

No info on directionality of radiation

(but maximum energy is defined, how?)

Max. Energy: \( E_e \)

\[ \text{Max. Energy: } E_e = \frac{\gamma m c^2}{2} \]

Decreasing energy

Elastic scattering:
Probability \( \sim Z^2/E_e^{-2} \)

Inelastic scattering: \( \nu \) release
Probability \( \sim Z^2 \)

High Z: Tungsten is a good target

Emission of x-rays II: What are Characteristic (fluorescent) X-rays?

Impacting e⁻ liberates inner shell e⁻:
1. Atom is excited (higher energy state)
2. Vacancy
3. Filled by outer shell electron (cascading)
4. Emission of characteristic x-ray

Auger emission
The excited atom can also reduce energy by liberating an additional e⁻ (Auger e⁻):